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Statistical approach for the first extraction of dipole dynamical polarizabilities from proton real Compton scattering data

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Summary. — In this work we describe a statistical technique that has been recently applied for the extraction of the proton scalar dipole dynamical polarizabilities from real Compton scattering data. The technique is based on the *simplex* minimization and on the *parametric bootstrap* and has several important advantages with respect to the standard χ^2 minimization technique, as, for example, the possibility to include in a straightforward way the systematic error in the minimization procedure.

1. – Polarizabilities in the proton

The polarizabilities of a composite system such as the nucleon are elementary structure constants, just as its size and shape. They can be accessed experimentally by Compton scattering processes. In the case of real Compton scattering (RCS), the incoming real photon deforms the nucleon, and by measuring the energy and angular distributions of the outgoing photon one can determine the induced current and magnetization densities. The global strength of these densities is characterized by the nucleon dipole and higher order (quasi-static) polarizabilities. By taking into account the spatial and time dependence of the electromagnetic field, the polarizabilities become energy dependent due to internal relaxation mechanisms, resonances and particle production thresholds in a physical system. This energy dependence is subsumed in the definition of dynamical polarizabilities, that parametrize the response of the internal degrees of freedom of a composite object to an external, real-photon field of arbitrary energy [1-3].

The definition of dynamical polarizabilities is given in terms of the multipole amplitudes $f^{l\pm}_{TT'}$, which correspond to the transition $Tl \to T'l'$, with $T, T' = E, M$ and $\pm$ giving the total angular momentum $j = l \pm 1/2$. In particular, for the scalar dipole electric $\alpha_{E1}(\omega)$ and magnetic $\beta_{M1}(\omega)$ dynamical polarizabilities one has the following

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combination of the $l = 1$ multipoles

$$\alpha_{E1}(\omega) \equiv \frac{2f_{EE}^{1+}(\omega) + f_{EE}^{1-}(\omega)}{\omega^2}, \qquad \beta_{M1}(\omega) \equiv \frac{2f_{MM}^{1+}(\omega) + f_{MM}^{1-}(\omega)}{\omega^2}, \tag{1}$$

where ω is the center of mass (c.m.) photon energy.

The extraction of the dynamical polarizabilities from RCS data is quite challenging. In this work we summarize the method proposed in Ref. [4] that allowed us to gain first insights on the scalar dipole dynamical polarizabilities (DDPs) from RCS data.

2. – Fitting technique

We adopt the following parametrization of the scalar DDPs

$$\begin{aligned} \alpha_{E1}(\omega) &= f_\alpha(\alpha_{E1}, \alpha_{E1,\nu}, \beta_{M1}, \beta_{M1,\nu}) + g_\alpha(\alpha_{E2}, \beta_{M2}, \gamma_i) + h_\alpha(h.o.), \\ \beta_{M1}(\omega) &= f_\beta(\alpha_{E1}, \alpha_{E1,\nu}, \beta_{M1}, \beta_{M1,\nu}) + g_\beta(\alpha_{E2}, \beta_{M2}, \gamma_i) + h_\beta(h.o.). \end{aligned} \tag{2}$$

In Eq. (2), $f_{\alpha,\beta} + g_{\alpha,\beta}$ correspond to the low-energy expansion (LEX) of the DDPs up to $\mathcal{O}(\omega^5)$, while $h_\alpha(h.o.)$ and $h_\beta(h.o.)$ parametrize the residual (higher-order $(h.o.)$) energy dependence beyond the LEX. The functions $g_{\alpha,\beta}$ depend on the higher-order static electric and magnetic polarizabilities α_{E2} and β_{M2}, and the static leading- and higher-oder spin polarizabilities γ_i. In our analysis, they are fixed using the predictions of fixed-t dispersion relations (DRs) for the higher-order polarizabilities [5-7], and the experimental values extracted in Ref. [8] for the leading-order static spin polarizabilities. Likewise, we use fixed-t DRs to calculate the functions $h_\alpha(h.o.)$ and $h_\beta(h.o.)$. The functions $f_{\alpha,\beta}(\alpha_{E1}, \alpha_{E1,\nu}, \beta_{M1}, \beta_{M1,\nu})$ correspond to the fitting functions, which depend on the leading order static polarizabilities α_{E1} and β_{M1} and the dispersive static polarizabilities $\alpha_{E1,\nu}$ and $\beta_{M1,\nu}$. In summary, we have four free parameters, i.e. $\alpha_{E1}, \alpha_{E1,\nu}, \beta_{M1}, \beta_{M1,\nu}$, with the additional constraint from the Baldin sum rule $\alpha_{E1} + \beta_{M1} = (13.8 \pm 0.4) \cdot 10^{-4}$ fm^3 [9], which reduces the number of fit parameters to three.

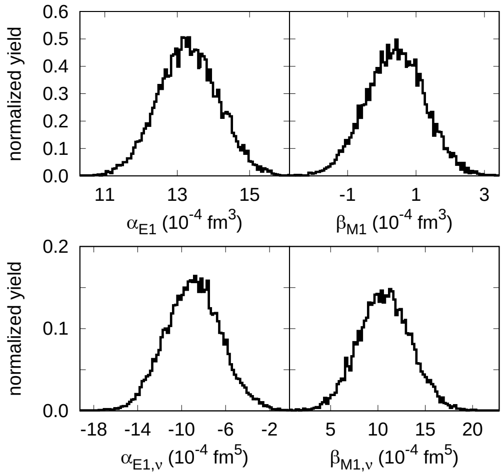
We used the data set of all the available experimental data for the unpolarized RCS cross section below pion-production threshold, as listed in Ref. [4]. First, we tried to apply the standard χ^2 minimization procedure with the Newton (gradient) method. However, we found that this method does not work due to (I) high correlation among the fitting parameters and (II) low sensitivity of the unpolarized RCS differential cross section to the DDPs coefficients. In particular, it was not possible to achieve the positive-definiteness condition of the covariance matrix.

In order to solve this problem, we combined the geometrical *simplex* method for the minimization in Minuit [10] and the *parametric bootstrap* [11], which is a Monte Carlo technique. The general idea is the approximation of the *true* (and unknown) probability distribution of the experimental datum with the (known) probability distribution given by the measured value with its statistical error. The only a priori assumption is the choice of the specific probability distribution associated to the experimental point.

Schematically, the bootstrap sampling can be written as

$$B_{i,j,k} = (1 + \delta_{j,k})(E_i + \gamma_{i,j}\sigma_i), \tag{3}$$

where E_i is the experimental value of the differential cross section, with a statistical error σ_i. In Eq. (3), the index i runs over the data points, j labels the number of replicas



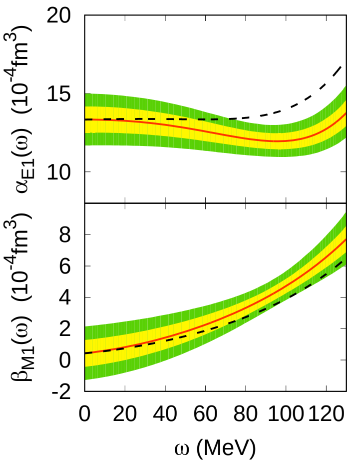


Fig. 1. – (a) Probability distributions for the fitted parameters of the scalar DDPs. (b) Results from the fit of the scalar DDPs as function of the c.m. photon energy ω: $\alpha_{E1}(\omega)$ on the top and $\beta_{M1}(\omega)$ on the bottom. The 68% (yellow) and 95% (green) *CL* areas include all the correlations between the parameters. The dashed lines represent the predictions from DRs [2].

and k indicates different data sub-sets. The number $\gamma_{i,j}$ is distributed according to a standard Gaussian $\mathcal{N}[0,1]$, while $\delta_{j,k}$ follows a box distribution $\mathcal{U}[-\Delta_k, \Delta_k]$, where Δ_k is the published systematic error for each data sub-set or, if more than one source for systematics is present, the product of all the contributions.

We then use our bootstrapped data $B_{i,j,k}$ for the usual definition of χ^2, that is minimized with the *simplex* technique: after every bootstrap cycle, we obtain the best values of the fitting parameters, thus reconstructing their probability distributions.

Our fitting technique has several advantages:

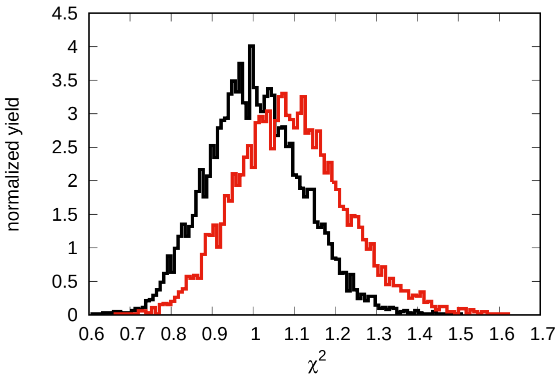


Fig. 2. – Probability distributions for the χ^2 obtained from our fitting technique in a two-parameters fit: the usual χ^2 distribution is obtained when we consider only statistical errors (black line), while a distortion is caused by the systematic sources (red line).

1. the straightforward inclusion of systematical errors with no modifications in the definition of the χ^2;

2. the *true* probability distributions for the fitted parameters (see Fig. 1(a)) with no a priori assumptions;

3. the calculation of the error propagation without using any approximated formulas;

4. the probability distributions of every functions of the fitted parameters (such as the DDPs or the differential cross section) which automatically include all the correlation terms.

In Fig. 1(b) we show our fit results for DDPs as function of the photon c.m. energy, with the 68% and 95% confidence level (CL) bands, in comparison with the predictions of DRs [2].

It is noteworthy to remark that the χ^2 reconstructed by our technique is *not distributed like a* χ^2 *distribution*, i.e. it is not the sum of squared Gaussian variables. This a consequence of the Monte Carlo sampling of Eq. (3) and reflects the biggest effect of the inclusion of systematical errors. As an example, we show in Fig. 2 the reconstructed probability distribution of the χ^2 in the case of a two-parameters fit, i.e. α_{E1} and β_{M1}, where the effect of the systematical errors is clearly visible.

3. – Conclusions

We have discussed a parametric bootstrap technique to analyze proton RCS data for the extraction of the scalar DDPs. We outlined several advantages of this technique and we refer to a forthcoming work [12] for a more comprehensive description of the statistical features of the method. This fitting procedure has never been applied so-far to analyze RCS data, and we plan to use it for a re-evaluation of the proton scalar dipole static polarizabilities [13] from the existing RCS data, using fixed-t DRs for the theoretical framework.

REFERENCES

[1] Griesshammer H. W., and Hemmert T. R., *Phys.Rev.C*, **65** (045207) 2002.
[2] Hildebrandt R. P., Griesshammer H. W., Hemmert T. R., and Pasquini B., *Eur. Phys. J. A*, **20** (293) 2004.
[3] Lensky V., McGovern J., and Pascalutsa V., *Eur. Phys. J.C*, **75** (604) 2015.
[4] Pasquini B., Pedroni P., and Sconfietti S.,*Phys.Rev. C*, **98** (015204) 2018.
[5] Pasquini B., Drechsel D., and Vanderhaeghen M., *Phys. Rev. C*, **76** (015203) 2007.
[6] Drechsel D., Gorchtein M., Pasquini B., and Vanderhaeghen M., *Phys.Rev.C*, **61** (015204) 1999.
[7] Holstein B. R., Drechsel D., Pasquini B., and M. Vanderhaeghen, *Phys.Rev.C*, **61** (034316) 2000.
[8] Martel P. P. et al. (A2), *Phys. Rev. Lett.*, **114** (112501) 2015.
[9] Patrignani C. et al. (Particle Data Group), *Chin. Phys. C*, **40** (100001) 2016.
[10] James F. and Roos M., *Comput. Phys. Commun.*, **10** (343) 1975.
[11] Davidson A. C. and Hinkley D. V., *Bootstrap Methods and their Application* (Cambridge University Press) 1997.
[12] Pasquini B., Pedroni P., Rotondi A., and Sconfietti S., in preparation.
[13] Pasquini B., Pedroni P., and Sconfietti S., in preparation.